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The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT
One promising avenue for the invention of new quantum algorithms is to focus on those that mimic physical processes. Our aim in this project is to identify physical quantum processes that can be reformulated to serve as quantum algorithms, with a particular focus on problems that are of intermediate difficulty in classical computation, including graph isomorphism, which is the problem of determining whether two graphs are related by a relabeling of the vertices. These problems are likely to be in the same class of problem difficulty as factoring, which has

15. SUBJECT TERMS
quantum algorithms, graph isomorphism, quantum random walks

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
UU	UU	UU	UU		Susan Coppersmith
					19b. TELEPHONE NUMBER: 608-262-8358

Report Title

Final Report for W911NF-09-1-0439, "Quantum Algorithms Based on Physical Processes."

ABSTRACT

One promising avenue for the invention of new quantum algorithms is to focus on those that mimic physical processes. Our aim in this project is to identify physical quantum processes that can be reformulated to serve as quantum algorithms, with a particular focus on problems that are of intermediate difficulty in classical computation, including graph isomorphism, which is the problem of determining whether two graphs are related by a relabeling of the vertices. These problems are likely to be in the same class of problem difficulty as factoring, which has proven to be amenable to attack by quantum computers. In this project we are investigating how the dynamical evolutions of different systems of interacting and noninteracting quantum particles can be exploited to attack the graph isomorphism problem. We have shown that seemingly minor changes in the algorithm can significantly affect its effectiveness in distinguishing nonisomorphic graphs, and we have also identified some restrictions on the power of the method when the number of particles in the walk is much less than the number of vertices.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
08/30/2011	2.00	John King Gamble, Mark Friesen, Dong Zhou, Robert Joynt, S. N. Coppersmith. Two-particle quantum walks applied to the graph isomorphism problem, Physical Review A, (05 2010): 0. doi: 10.1103/PhysRevA.81.052313
11/28/2013	5.00	John King Gamble, Mark Wellons, Eric Bach, Mark Friesen, Robert Joynt, S. N. Coppersmith, Kenneth Rudinger. Noninteracting multiparticle quantum random walks applied to the graph isomorphism problem for strongly regular graphs, Physical Review A, (08 2012): 0. doi: 10.1103/PhysRevA.86.022334
11/28/2013	6.00	John King Gamble, Kenneth Rudinger, Eric Bach, Mark Friesen, Robert Joynt, S. N. Coppersmith, Adam Frees. Power-law scaling for the adiabatic algorithm for search-engine ranking, Physical Review A, (09 2013): 0. doi: 10.1103/PhysRevA.88.032307
11/28/2013	7.00	Kenneth Rudinger, John King Gamble, Eric Bach, Mark Friesen, Robert Joynt, S. N. Coppersmith. Comparing Algorithms for Graph Isomorphism Using Discrete- and Continuous-Time Quantum Random Walks, Journal of Computational and Theoretical Nanoscience, (07 2013): 0. doi: 10.1166/jctn.2013.3105

TOTAL: **4**

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Susan Coppersmith, "Physical intuition and quantum algorithms," invited talk, Conference on Imagery for Insight into Material Structure (symposium in honor of Thomas A. Witten), University of Chicago, November 2013

Susan Coppersmith, "Investigation of a quantum adiabatic algorithm for search engine ranking," invited talk, Rutgers Statistical Mechanics Conference, Rutgers University, May 2013

Adam Frees, John King Gamble, Kenneth Rudinger, Eric Bach, Mark Friesen, Robert Joynt, and S. N. Coppersmith, "Power law scaling for the adiabatic algorithm for search engine ranking," APS March Meeting, Baltimore, MD, March 2013

Kenneth Rudinger, John King Gamble, Eric Bach, Mark Friesen, Robert Joynt, and S. N. Coppersmith, "Using the graph isomorphism problem to probe differences between discrete- and continuous-tme quantum random walks," APS March Meeting, Baltimore, MD, March 2013

Susan N. Coppersmith “Quantum random walks of interacting particles and the graph isomorphism problem,” NASA Quantum Technologies Conference, Monterey, California, January 2012; also colloquia with same title presented at Brandeis University (3/12) and Ohio University (5/12).

Kenneth Rudinger; “Multiparticle Quantum Walks and the Graph Isomorphism Problem,” 2/29/12; American Physical Society March Meeting; Boston, MA

S.N. Coppersmith, “Quantum random walks of interacting particles and the graph isomorphism problem,” Conference on “Sphere Packing and Amorphous Materials," ICTP, Trieste, Italy, July 2011 (invited)

John King Gamble, Mark Friesen, Dong Zhou, Robert Joynt, and S.N. Coppersmith, “Two-particle quantum walks applied to the graph isomorphism problem,” American Physical Society March meeting, March 2011

Mark Wellons, John Gamble, Eric Bach, Mark Friesen, Robert Joynt, Kenneth Rudinger, Dong Zhou, and Susan Coppersmith, “Numerical investigations of quantum walks with hard-core bosons and the graph isomorphism problem,” American Physical Society March meeting, March 2011

Kenneth Rudinger, John King Gamble, Mark Wellons, Mark Friesen, Dong Zhou, Eric Bach, Robert Joynt, and S.N. Coppersmith, “Quantum random walks of non-interacting bosons on strongly regular graphs,” American Physical Society March meeting, March 2011

Number of Presentations: 12.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
08/02/2010	1.00	John King Gamble, Mark Friesen, Dong Zhou, Robert Joynt, S. N. Coppersmith. Two-particle quantum walks applied to the graph isomorphism problem, Physical Review A (05 2010)
08/03/2012	3.00	Kenneth Rudinger, John King Gamble, Mark Wellons, Eric Bach, Mark Friesen, Robert Joynt, S. N. Coppersmith. Non-interacting multi-particle quantum random walks applied to the graph isomorphism problem for strongly regular graphs, arXiv:1206.2999 (06 2012)
08/03/2012	4.00	Kenneth Rudinger, John King Gamble, Eric Bach, Mark Friesen, Robert Joynt, S. N. Coppersmith. Comparing algorithms for graph isomorphism using discrete- and continuous-time quantum random walks, arXiv:1207.4535 (07 2012)
11/28/2013	8.00	Eric Bach, Jonathan Sorenson. Approximately counting semismooth integers, To appear in ISSAC 2013, Boston MA (01 2013)
TOTAL:		**4**

Number of Manuscripts:

Books

Received Paper

TOTAL:

Patents Submitted

Patents Awarded

Awards

Susan Coppersmith:
Vilas Professorship, University of Wisconsin-Madison
Outstanding Referee, Physical Review and Physical Review Letters (2012)
Scientific Advisory Board, Simons Foundation (2013-2016)
Executive Line, Section on Physics, American Association for the Advancement of Science (2013-2016)

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Kenneth Rudinger	0.60	
Mark Wellons	0.06	
FTE Equivalent:	**0.66**	
Total Number:	**2**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Eric Bach	0.11	
Susan Coppersmith	0.10	Yes
Robert Joynt	0.09	
FTE Equivalent:	**0.30**	
Total Number:	**3**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME
Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
Mark Friesen	0.11
FTE Equivalent:	**0.11**
Total Number:	**1**

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Statement of problem studied:
Our aim in this project is to identify physical quantum processes that can be reformulated to serve as quantum algorithms, with a particular focus on problems that are of intermediate difficulty in classical computation, including graph isomorphism, the shortest lattice vector problem, and the matroid parity problem. These problems are likely to be in the same class of problem difficulty as factoring, which has proven to be amenable to attack by quantum computers. We investigated how the dynamical evolutions of different systems of interacting and noninteracting quantum particles can be exploited to attack the graph isomorphism problem, in which one is trying to distinguish graphs that are very similar but not precisely equivalent. We have shown that seemingly minor changes in the algorithm can significantly affect its effectiveness in distinguishing nonisomorphic graphs, and we have also identified some restrictions on the power of the method when the number of particles in the walk is much less than the number of vertices. We also studied the properties of directed graphs, where the links have prescribed directionality, to investigate whether a quantum adiabatic algorithm can be used to compute efficiently the "google vector" of the associated network.

Summary of most important results:
The main thrust of the work was an investigation of the ability of systems of both interacting and noninteracting Bosons to distinguish nonisomorphic graphs.
1) We showed that quantum walks of two hard-core Bosons can distinguish all pairs of nonisomorphic strongly regular graphs tested (we tested graph pairs with up to 64 vertices) [1].
2) We showed that quantum walks of three or more noninteracting Bosons could distinguish some but not all pairs of strongly regular graphs, and investigated in detail the mechanisms by which noninteracting quantum particles could distinguish graphs that classical particles could not [2].
3) We investigated in detail differences between different formulations of quantum walks in discrete and continuous time [3]. While we believe that the asymptotic performance of discrete- and continuous-time walks will be the same, there are graph pairs for which discrete-time walks can distinguish walks that continuous-time walks cannot. This phenomenon is important to understand, because numerical investigations always use a finite number of graph pairs. We identified the features of discrete-time walks that lead to these differences.
4) Google computes the eigenvector of the largest eigenvalue of the "Google matrix" and uses it in its algorithm for generating search results. Garnerone et al. [4] present evidence that this eigenvector can be calculated efficiently on an adiabatic quantum computer on Google matrices that encode the connectivity of graphs that have power-law degree distributions. We investigated the "Google matrices" of a broad variety of graphs with the power-law degree distributions typical of the internet and found that the method used to construct the graph affects how the eigenvalue gap that governs the speed of the quantum algorithm varies with graph size [5]. This result means that more understanding of the connectivity of the internet is needed to know how the adiabatic quantum algorithm will scale with matrix size.

References:

[1] John King Gamble, Mark Friesen, Dong Zhou, Robert Joynt, S. N. Coppersmith, Phys. Rev. A 81, 052313 (2010), preprint arXiv:1002.3003.

[2] Kenneth Rudinger, John King Gamble, Mark Wellons, Eric Bach, Mark Friesen, Robert Joynt, S. N. Coppersmith, Phys. Rev. A 86, 022334 (2012), preprint arXiv:1206.2999.

[3] Kenneth Rudinger, John King Gamble, Eric Bach, Mark Friesen, Robert Joynt, S. N. Coppersmith, J. Comput. Theor. Nanos. 10 (7), pp. 1653-1661 (2013), preprint arXiv:1207.4535

[4] Silvano Garnerone et al., Phys. Rev. Lett. 108, 230506 (2012).

[5] Adam Frees, John King Gamble, Kenneth Rudinger, Eric Bach, Mark Friesen, Robert Joynt, S. N. Coppersmith, preprint arXiv:1211.2248.

Technology Transfer